



*The members of the Pythagorean society referred to each other as "brother".

Don't worry! Let's have a look at the table of squares.

Numbers	Squares	Double squares
1	1	2
2	4	8
3	9	18
4	16	32

Brothers! See, there is no square number in the third column.

Look! These numbers continue forever.

Don't you think that eventually one of the numbers of the third column will appear in the second column?

Haven't you read that everything which is measurable can be measured by numbers?

I have read that, but I don't know how it works here.

Okay, let me divide this matchstick into two equal parts. Do you know a number that can express each part?

Is there really such a number?

Yes! Very naturally! That is one part out of two parts; "one-half".

This is the number we use to express these parts:

"One-half"

I understand this. But I don't understand how these numbers solve my problem.

Think of a one-by-one square. Now, you can measure the side of the square with two of these small pieces, and to measure the diameter...

Yes, yes! Three! Since for a two-by-two square we need three matchsticks...

Yes! Three "one-halves", which we call "three halves".

Of course, three halves is bigger than the diameter of a one-by-one square.

So, even these numbers, that I don't know what we call them yet, don't work.

Don't you understand that every measurable thing can be measured by our numbers!

Yes, "one-half" didn't work, but certainly there's another fraction* that works, for example "one-fifth".

Is there?

Yes, there is! Providing that you don't just try out a five-by-five square and say it can't be done. Remember that every measurable thing...

It's a bit short... My God!...Always!

Let's drown him in the sea!

Yes, he may plant seeds of doubt in the minds of others.

And they threw him in the sea.

Since then he turned "ir-rational".



Later on some bigger brothers found that the previous big brothers were mistaken. In fact, our little brother was right.

None of the numbers of the third column will ever appear in the second column. So it is impossible to express the diameter of a one-by-one square with fractions.

To do this, we need a new kind of number that, in honor of our little brother, is called "irrational number"!

The diameter in our story is just one of them which is called the square root of 2, and is written as $\sqrt{2}$

*They were so angry that they forgot to mention what they meant by fraction is numbers like one-half and two-thirds