Concept Image–Concept Definition in the Era of AI

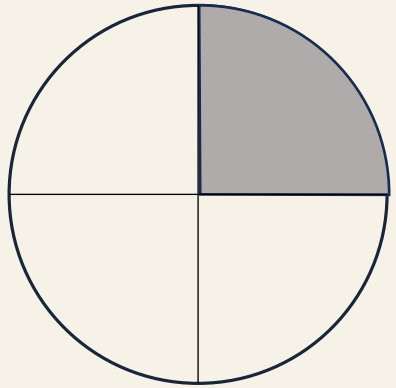
A light look back at Tall–Vinner, Edwards–Ward, and a new tension in mathematical learning

Amir H. Asghari

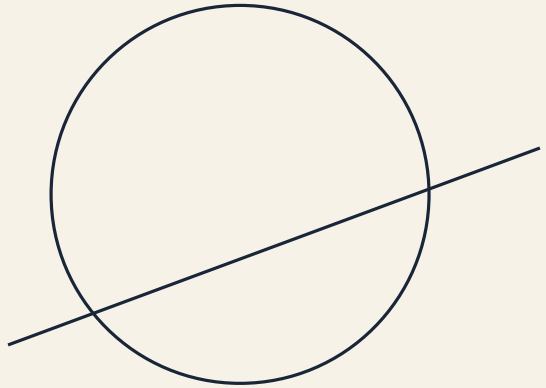
Approaching the second anniversary of David Tall's passing, 15 July 2024

Two different circles, two different recalls

What fraction of the circle is shaded?



In how many points does the line cross the circle?



everyday image
(circle as disk)

formal definition
(points from a centre)

Expert readers switch almost invisibly between these recalls. Learners may not.

Tall and Vinner: the original distinction

1981

Concept image

Mental pictures, associated properties, and processes.

Concept definition

Words used to specify the concept.

Since then, “students’ concept image of …” has become a familiar starting phrase across mathematics education research.

Edwards and Ward

2004

Ward’s bet

Terms such as group and coset are “not so loaded”. Surely, definitions will do more work.

But students still imported familiar routines.

Surprise 1 Students do not know that a definition decides what the word means.

Surprise 2 students may state a definition correctly, yet not work from it

Surprise 3 even “no alternative” leaves room for familiar images and routines

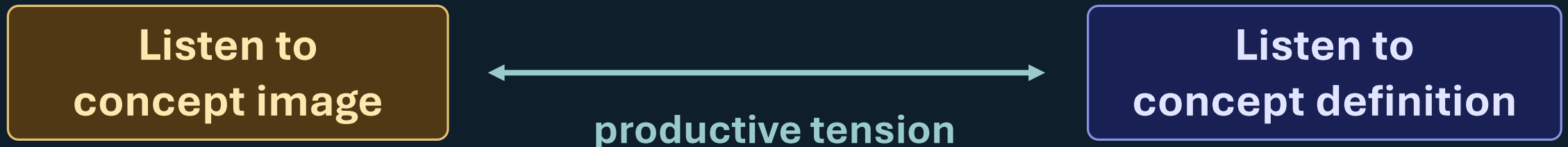
The surprise is not that students lack definitions.
It is that definitions do not yet function for them as tools for deciding meaning and action.

The disciplinary role of definitions

1	Definitions determine meaning and usage.	→	1 Yet we develop ways to lessen their cognitive demands.
2	Definitions need not articulate familiar images.	→	2 They can bring new objects into being by saying how they behave.
3	Definitions and images work together	→	3 Concept definitions give control; Concept images ease action.

The issue is not simply whether students use definitions.
It is how they learn to regulate the relation between words, images, and action.

From “definition versus image” to interplay



Expert practice is not the replacement of concept image by concept definition.

It is knowing when, why, and how each should constrain the other.

A third voice: public concept image

Teachers, books, diagrams, examples, gestures, and now LLMs do not only offer definitions.

They also offer images of how the concept is meant to be seen and used.

LLMs have access to public concept images and can turn them, with ease, into words, diagrams, and analogies.

AI changes the landscape

Public mathematical language and images are now easy to summon.

Before

Definitions were often hard to access, articulate, or remember.
Examples were limited.
Analogies depended on a teacher's imagination and language.

Now

LLMs can produce definitions, reword them, supply examples in seconds, and suggest plausible analogies.

Concept definition has become easy to access as language.
Public concept image has become abundant.

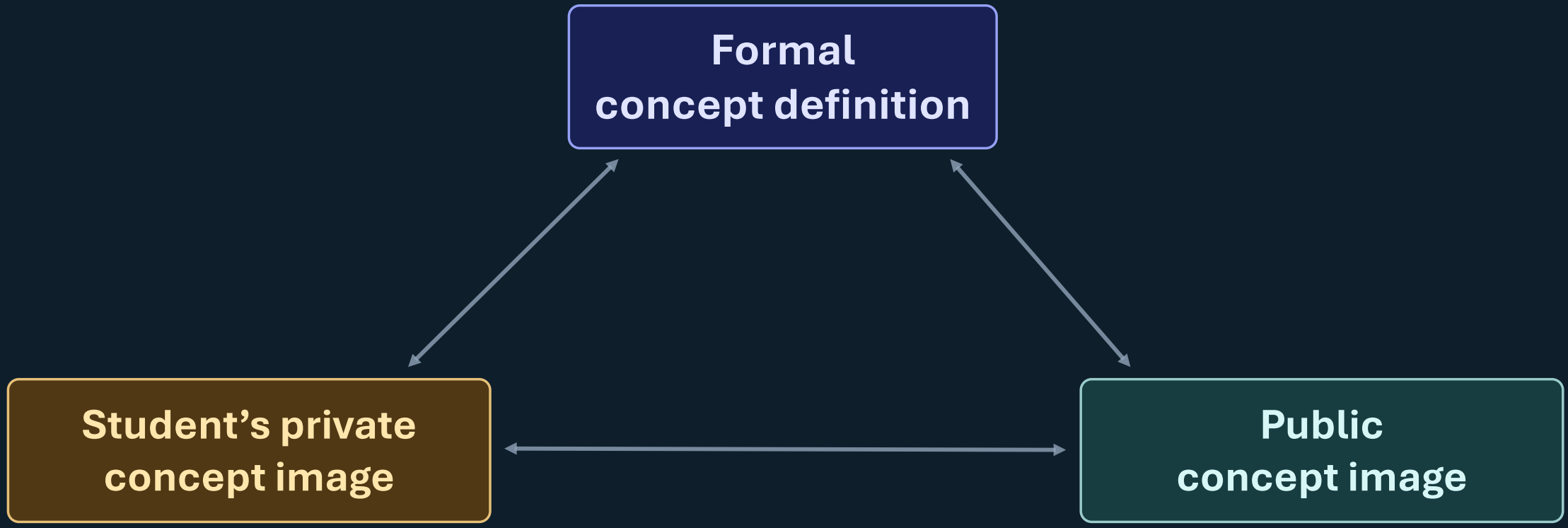
Yet the bottleneck remains private concept image

AI can suggest words.
AI can suggest diagrams.
AI can even suggest metaphors.

But it cannot simply install a learner's own flexible, usable concept image.

The work of meaning-making still belongs to the learner.

The triangle of concept formation



The task is to form private concept images, understand public ones, and keep both answerable to concept definitions.

Teaching by engaging the third voice

Use familiar tools with an added purpose: make the image–definition relation visible.

Non-example-looking examples

Ask which image is being challenged.

Example-looking non-examples

Ask what in the example invites this concept image.

Proof-generated definitions

Ask what the definition must do.

Definitions in a wider scheme

Ask when a local image stops helping.

Students need to learn not to outsource thinking to AI, but to engage with it as one public voice in their own concept formation.

Thank you

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Closing question

When AI speaks loudly, how do we help students hear their own mathematical voice?

Selected sources

- Tall & Vinner (1981), Concept image and concept definition
- Edwards & Ward (2004), Student (mis)use of mathematical definitions
- Asghari, RUME Reviews contribution to The American Mathematical Monthly